

$$\begin{aligned} 1. \quad & 4x - y + 2z = 2 \\ & -3x + y - 4z = -1 \\ & x + 4z = 1 \end{aligned}$$

$$\left| \begin{array}{ccc|c} 4 & -1 & 2 & 2 \\ -3 & 1 & -4 & -1 \\ 1 & 0 & 4 & 1 \end{array} \right|$$

$$\frac{R_1}{4} \rightarrow R_1$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ -3 & 1 & -4 & -1 \\ 1 & 0 & 4 & 1 \end{array} \right|$$

$$3R_1 + R_2 \rightarrow R_2$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{4} & -\frac{5}{2} & \frac{1}{2} \\ 1 & 0 & 4 & 1 \end{array} \right|$$

$$4R_2 \rightarrow R_2$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -10 & 2 \\ 1 & 0 & 4 & 1 \end{array} \right|$$

$$R_3 - R_1 \rightarrow R_3$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -10 & 2 \\ 0 & \frac{1}{4} & \frac{7}{2} & \frac{1}{2} \end{array} \right|$$

$$4R_3 \rightarrow R_3$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -10 & 2 \\ 0 & 1 & 14 & 2 \end{array} \right|$$

$$R_3 - R_2 \rightarrow R_3$$

$$\left| \begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -10 & 2 \\ 0 & 0 & 24 & 0 \end{array} \right|$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{7}{2} = \frac{1}{2}$$

$$3(1) - 3 = 0 \quad 3(-\frac{1}{4}) + 1 = -\frac{3}{4} + \frac{4}{4} = \frac{1}{4}$$

$$3(\frac{1}{2}) - 4 = \frac{3}{2} - \frac{8}{2} = -\frac{5}{2}$$

$$3(\frac{1}{2}) - 1 = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

$$4(\frac{1}{2}) = 1$$

$$4(-\frac{5}{2}) = -10$$

$$4(\frac{1}{2}) = 2$$

$$0 - (-\frac{1}{4}) = \frac{1}{4}$$

$$4 - \frac{1}{2} = \frac{8}{2} - \frac{1}{2} = \frac{7}{2}$$

$$1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$$

$$4(\frac{7}{2}) = 14$$

$$4(\frac{1}{2}) = \frac{2}{2} = 1$$

$$14 - (-10) = 24$$

$$2 - 2 = 0$$

$$24z = 0$$

$$z = 0$$

$$y - 10z^6 = 2$$

$$y = 2$$

$$\begin{matrix} x=1 \\ y=2 \\ z=0 \end{matrix}$$

$$x - \frac{1}{4}y + \frac{1}{2}z^0 = \frac{1}{2}$$

$$x - \frac{1}{4}(2) = \frac{1}{2}$$

$$x - \frac{1}{2} = \frac{1}{2}$$

$$x = \frac{1}{2} + \frac{1}{2} = 1$$

$$\begin{aligned} 2. \quad & 2x + 3y + 4z = -6 \\ & -x + 4y - 6z = 6 \\ & 3x - 2y + 2z = 2 \end{aligned}$$

$$\left| \begin{array}{ccc|c} 2 & 3 & 4 & -6 \\ -1 & 4 & -6 & 6 \\ 3 & -2 & 2 & 2 \end{array} \right|$$

$$\frac{R_1}{2} \rightarrow R_1$$

$$\left| \begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ -1 & 4 & -6 & 6 \\ 3 & -2 & 2 & 2 \end{array} \right|$$

$$\frac{4}{2} = 2, \quad \frac{-6}{2} = -3$$

$$R_2 + R_1 \rightarrow R_2$$

$$\left| \begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 11/2 & -4 & 3 \\ 3 & -2 & 2 & 2 \end{array} \right|$$

$$4 + \frac{3}{2} = \frac{8}{2} + \frac{3}{2} = \frac{11}{2}$$

$$-6 + 2 = -4$$

$$6 - 3 = 3$$

$$\frac{2}{11} R_2 \rightarrow R_2$$

$$\left| \begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 3 & -2 & 2 & 2 \end{array} \right|$$

$$\frac{2}{11}(-4) = -\frac{8}{11}$$

$$\frac{2}{11}(3) = \frac{6}{11}$$

$$3R_1 - R_3 \rightarrow R_3$$

$$\left| \begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 0 & 13/2 & 4 & -11 \end{array} \right|$$

$$3\left(\frac{3}{2}\right) - (-6) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$$

$$3(2) - 2 = 6 - 2 = 4$$

$$3(-3) - 2 = -9 - 2 = -11$$

$$\frac{13}{2} R_2 - R_3 \rightarrow R_3$$

$$\left| \begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 0 & 0 & 8/11 & 160/11 \end{array} \right|$$

$$\frac{13}{2}\left(-\frac{8}{11}\right) - 4 = \frac{52}{11} - \frac{44}{11} = \frac{8}{11}$$

$$\frac{13}{2}\left(\frac{6}{11}\right) - (-11) = \frac{39}{11} + 11 = \frac{39}{11} + \frac{121}{11} = \frac{160}{11}$$

$$\begin{aligned} 2. \quad & 2x + 3y + 4z = -6 \\ & -x + 4y - 6z = 6 \\ & 3x - 2y + 2z = 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & -6 \\ -1 & 4 & -6 & 6 \\ 3 & -2 & 2 & 2 \end{array} \right]$$

$$\frac{F_1}{2} \rightarrow F_1$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ -1 & 4 & -6 & 6 \\ 3 & -2 & 2 & 2 \end{array} \right]$$

$$F_1 + F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 11/2 & -4 & 3 \\ 3 & -2 & 2 & 2 \end{array} \right]$$

$$3/2 + 4 = \frac{3}{2} + \frac{8}{2} = \frac{11}{2}$$

$$2 - 6 = -4$$

$$-3 + 6 = 3$$

$$\frac{2}{11} F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 3 & -2 & 2 & 2 \end{array} \right]$$

$$-4 \left(\frac{2}{11} \right) = -\frac{8}{11}$$

$$3 \left(\frac{2}{11} \right) = \frac{6}{11}$$

$$3F_1 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 0 & 13/2 & 4 & -11 \end{array} \right]$$

$$3 \left(\frac{3}{2} \right) - (-2) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$$

$$3(2) - 2 = 6 - 2 = 4$$

$$3(-3) - 2 = -9 - 2 = -11$$

$$\frac{2}{13} F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 0 & 1 & 8/13 & -22/13 \end{array} \right]$$

$$\frac{2}{13} (4) = \frac{8}{13}$$

$$\frac{2}{13} (-11) = -\frac{22}{13}$$

$$F_2 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 3/2 & 2 & -3 \\ 0 & 1 & -8/11 & 6/11 \\ 0 & 0 & -192/143 & -320/143 \end{array} \right]$$

$$-\frac{8}{11} - \frac{8}{13} = \frac{-8(11) - 8(13)}{143} = -\frac{192}{143}$$

$$\frac{6}{11} + \left(-\frac{22}{13} \right) = \frac{6(13) + 11(-22)}{143} = -\frac{320}{143}$$

$$-\frac{192}{143} z = -\frac{320}{143}$$

$$-192z = -320$$

$$z = \frac{-320}{-192}$$

$$y - \frac{2}{11} z = \frac{6}{11}$$

$$y = \frac{6}{11} + \frac{2}{11} \left(\frac{-320}{-192} \right)$$

$$y = \frac{-2}{3}$$

$$x + \frac{3}{2}y + 2z = -3$$

$$x = -3 + \frac{3}{2} \left(\frac{-2}{3} \right) - 2 \left(\frac{-320}{-192} \right)$$

$$x = \frac{4}{3}$$

$$\begin{aligned} 3. \quad & x + 2y - z = 1 \\ & -3x - 5y + 2z = -5 \\ & 2x + 6y + 3z = -2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ -3 & -5 & 2 & -5 \\ 2 & 6 & 3 & -2 \end{array} \right]$$

$$-\frac{F_2}{3} \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 1 & 5/3 & -2/3 & 5/3 \\ 2 & 6 & 3 & -2 \end{array} \right]$$

$$F_1 - F_2 \rightarrow F_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1/3 & -1/3 & 2/3 \\ 2 & 6 & 3 & -2 \end{array} \right]$$

$$2 - \frac{5}{3} = \frac{6}{3} - \frac{5}{3} = \frac{1}{3}$$

$$-1 - (-\frac{2}{3}) = -1 + \frac{2}{3} = -\frac{3}{3} + \frac{2}{3} = -\frac{1}{3}$$

$$1 - \frac{5}{3} = \frac{3}{3} - \frac{5}{3} = -\frac{2}{3}$$

$$3F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -2 \\ 2 & 6 & 3 & -2 \end{array} \right]$$

$$2F_1 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & -2 & -5 & 4 \end{array} \right]$$

$$2(2) - 6 = 4 - 6 = -2$$

$$2(-1) - 3 = -2 - 3 = -5$$

$$2(1) - (-2) = 2 + 2 = 4$$

$$2F_2 + F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -7 & 0 \end{array} \right]$$

$$2(-1) - 5 = -2 - 5 = -7$$

$$2(-2) + 4 = -4 + 4$$

$$\begin{aligned} -7z &= 0 \\ z &= 0 \end{aligned}$$

$$\begin{aligned} x - z &= -2 \\ y &= -2 \end{aligned}$$

$$x + 2y - z = 1$$

$$x + 2(-2) - 0 = 1$$

$$x - 4 = 1$$

$$x = 1 + 4$$

$$x = 5$$

$$4. \quad x+y+z = 0$$

$$2x-y+2z = 7$$

$$-3x+2y+7z = 5$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & 2 & 7 \\ -3 & 2 & 7 & 5 \end{array} \right]$$

$$2F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 3 & 0 & -7 \\ -3 & 2 & 7 & 5 \end{array} \right]$$

$$\frac{F_2}{3} \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -\frac{7}{3} \\ -3 & 2 & 7 & 5 \end{array} \right]$$

$$3F_1 + F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -\frac{7}{3} \\ 0 & 5 & 10 & 5 \end{array} \right]$$

$$5F_2 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -\frac{7}{3} \\ 0 & 0 & -10 & -\frac{50}{3} \end{array} \right]$$

$$y = -\frac{7}{3}$$

$$x+y+z = 0$$

$$x = -y-z$$

$$x = -\left(y+z\right)$$

$$x = -\left(-\frac{7}{3} + \frac{5}{3}\right)$$

$$x = -\left(-\frac{2}{3}\right)$$

$$x = \frac{2}{3}$$

$$2x - z - y = -2z$$

$$2x - y + 2z = 7$$

$$2y + 7z - 5 = 3x$$

$$-3x + 2y + 7z = 5$$

$$2(1) - (-1) = 2 + 1 = 3$$

$$2(1) - 2 = 0$$

$$2(0) - 7 = -7$$

$$3(1) + 2 = 5$$

$$3(0) + 7 = 10$$

$$3(0) + 5 = 5$$

$$5(1) - 5 = 0$$

$$5(0) - 10 = -10$$

$$5\left(-\frac{7}{3}\right) - 5$$

$$= -\frac{35}{3} - \frac{15}{3}$$

$$= -\frac{50}{3}$$

$$-10z = -\frac{50}{3}$$

$$z = \frac{5}{3}$$

$$\begin{aligned} 5. \quad w - x - 2y &= 0 \\ 2w - x + y &= 0 \\ 2w - 3x - y &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 2 & -1 & 1 & 0 \\ 2 & -3 & -1 & 0 \end{array} \right] \quad \begin{aligned} 2(-1) - (-1) &= -2+1 = -1 \\ 2(2)+1 &= -4+1 = -3 \end{aligned}$$

$$2F_1 - F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & -1 & -5 & 0 \\ 2 & -3 & -1 & 0 \end{array} \right]$$

$$-F_2 \rightarrow F_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 2 & -3 & -1 & 0 \end{array} \right]$$

$$2(-1) - (-3) = -2+3 = 1$$

$$2(-2) - (-1) = -4+1 = -3$$

$$-2F_1 - F_3 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 1 & -3 & 0 \end{array} \right]$$

$$F_3 - F_2 \rightarrow F_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & -8 & 0 \end{array} \right]$$

$$-3 - 5 = -8$$

$$-8z = 0$$

$$z = 0$$

$$y + 5z = 0$$

$$y = 0$$

$$x - y - 2z = 0$$

$$x = 0$$